Analysis of Real and Complex Modes of Grounded Slab with the Transverse Resonance Method

Riham Borghol
University of Tunis El Manar, National Engineering School of Tunis, LR99ES21 Lab. Sys' Com, ENIT
Tunis, Tunisia
e-mail: borgholriham@gmail.com

Aguili taoufik
University of Tunis El Manar, National Engineering School of Tunis, LR99ES21 Lab. Sys' Com, ENIT
Tunis, Tunisia
e-mail: Taoufik.aguili@gmail.com

***Abstract*— In this work, we focus our attention on dispersion characteristics of Transverse Electric (TE) and Transverse Magnetic (TM) surface modes, leaky modes and volume modes of double-positive grounded slab. We have analyzed the dispersion using the Transverse Resonance Method (TRM). The simple method represents the boundary conditions of the grounded slab by an equivalent circuit. It reveals that this structure, though it contains several types, just the ordinary type exists for surface modes and just the improper type exists for leaky modes. Numerical results illustrate the properties and the existence conditions for these modes.**

***Keywords*— Grounded SLAB; surface modes; volume modes; leaky modes; Transverse Resonance Method.**

I. Introduction

A few years ago, the miniaturization of antennas leads to the appearance of new modes which are the surface modes. These modes have received considerable attention in the scientific community. Surface modes are used in different fields for many applications, such as the surface Radar [1], the surface wave filters [2], the surface plasmons for biodetection[3] as well as the seismic imaging with surface wave [2]. In electromagnetic, these modes can approach the antenna elements and this rapprochement generates the coupling phenomenon. Therefore, they provide a new way to miniaturization [4]. Surface modes exist at a discontinuity interface between two different mediums. At this interface, leaky modes can also exist. Usually, leaky modes are excited when perturbing a surface mode or a guided mode propagating in an open waveguide with proper periodic corrugations or modifications [5]. These Leaky modes propagate faster than the speed of the light and, at the same time, they leak energy along the interface. Leaky modes are used for applications in the millimeter-wave ranges.

Based on the modal analysis for double-positive grounded slab, the present authors have revealed that only surface and leaky modes can exist for TE and TM polarization [6][7],but they did not demonstrated the existence of volume mode which is identified as the magnetostatic wave. Volume mode is characterized by the limited field to structure and no energy is lost from it.

In this work, we investigate the propagation of real modes (i.e., surface, volume) [8][9] and complex modes (i.e., leaky) [5][10] supported by a slab placed on a perfectly conducting ground plane (grounded slab). In Section 2, we will present the other existing works and we will compare them with our work. In Section 3, we will analyse the dispersion characteristics of TE and TM modes based on a general method called the Tranverse Resonance Method (TRM) [11][12]. In Section 4, we will discuss the results obtained from the MATLAB simulations. In Section 5, we will conclude this paper.

II. STATE OF THE ART

In 2003, Paolo Baccarelli has worked on the propagation of surface waves in a particular structure double-negative metamaterial grounded slab [8]. In 2004, the same author has presented the dispersion and radiation properties of leaky waves on the same structure [7]. He has demonstrated that leaky waves of only the proper type exist in double-negative grounded slab, while proper or improper leaky waves exist in single negative grounded slab. Paolo Baccarelli has investigated in his work only the dispersion of surface and leaky modes on single- and double-negative grounded slabs.

Our work focuses on the dispersion of surface, volume and leaky modes in double-positive grounded slab. In literature, there are many techniques through which we can realize this analysis such as the analytical technique dyadic Green's function [13][14] which can be estimated to complex forms depending on how the material is described macroscopically. Another technique is postulating fields on one side of the discontinuity and using Snel's law of reflection and refraction [15]. In our study, we use The TRM. This approach allows deriving the dispersion properties of real and complex modes for open structure in a simple way. It is also useful to describe the open structure with an equivalent circuit parameters as admittance or reflection coefficient. This circuit model permits to find in a simple way the dispersion equation for the two polarizations TE and TM. The resolution of these two equations permits to know the modes in this structure and their existence conditions.

III. ANALYSIS

Let us consider a grounded slab characterized by a lossless dielectric medium of height d and relative constitutive parameters ε_r and $\mu_r = \mu_0$. It is infinite along the two directions x and z, as shown in Fig.1.

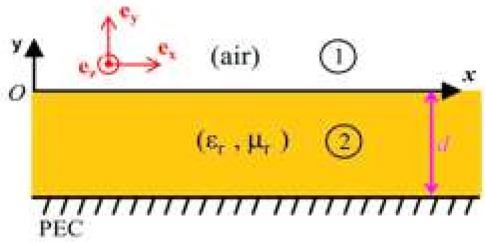

Figure 1. Schematic illustration of infinity grounded slab. The dielectric slab is characterized by a relative permittivity ε_r and permeability μ_r.

Taking into consideration the assumption that no variation along the x direction and that the electromagnetic field depends on time and the longitudinal coordinate as $e^{-j(\omega t + k_z z)}$, the real and complex modes can be separated into two polarizations TE modes with $E_y = E_z = 0$ and TM modes with $H_y = H_z = 0$. The field related to such structure can be travel in an equivalent transmission line model illustrated in Fig. 2. This model describes the transverse discontinuity with equivalent network parameters to simplify the resolution of the Maxwell equations using the TRM.

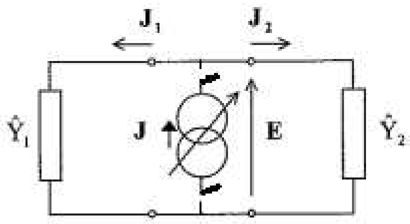

Figure 2. Equivalent circuit used to illustrate the transverse Resonance Method.

The symbolic representation of the two homogeneous media (air, dielectric) separated by the discontinuity plan (xoz) is given by two equivalent admittances Y_1 and Y_2. The equivalent admittance $Y_2(k_z, k_0)$ of the dielectric slab is a function of the transverse propagation constant k_z and the free space propagation constant k_0. Consequently, it is then possible to find the dispersion equation by using the univalent circuit. The variable j, which is a virtual current source, is zero at the discontinuity plan. It is equivalent to an open circuit. E is a dual grader [14].

According to Kirchhoff's current and Ohm's law, we deduce :

$$(Y_1 + Y_2) = 0 \tag{1}$$

Where $Y_1 = y_{1M}^{\alpha}$ and $Y_2 = y_{2M}^{\alpha} \coth(k_{y2} d)$.

$\alpha = TE, TM$ indicate the polarization supported by the structure. The expression of the relevant characteristic admittances for the two polarizations areas are the following [15]:

$$y_{1M}{}^{TE} = \frac{k_{y1}}{j\omega\mu_0} \tag{2}$$

$$y_{1M}^{TM} = \frac{j\omega\varepsilon_0}{k_{y1}} \tag{3}$$

$$y_{2M}{}^{TE} = \frac{k_{y2}}{j\omega\mu_0} \tag{4}$$

$$y_{2M}{}^{TM} = \frac{j\omega\varepsilon_0\varepsilon_r}{k_{y2}} \tag{5}$$

Where k_{y1} and k_{y2} are the transverse wavenumbers.

The use of the equivalent circuit model allows us to derive the dispersion relation of the structure for TE and TM modes:

TE:

$$k_{y2}\coth(k_{y2}\, d) + k_{y1} = 0 \tag{6}$$

TM:

$$k_{y2}\, th(k_{y2}\, d) + k_{y1}\, \varepsilon_r = 0 \tag{7}$$

The resolution of these two equations gives the transverse propagation constant k_z of real and complex modes. The real modes propagate in z direction with a real propagation constant $k_z = \beta_z$ and the complex modes propagate with a complex propagation constant $k_z = \beta_z - j\alpha_z$ [16].

A. Surface Modes

1) TM Polarization

In the TM polarization, the magnetic fields can be written in the two regions as:

$$\begin{cases} H_{x1} = C_1 e^{-jk_z z} e^{-k_{y1} y} \\ H_{x2} = C_1 e^{-jk_z z} \cosh(k_{y2}(y+d)) \end{cases} \tag{8}$$

Surface mode propagates without radiation along the structure. There is no attenuation of the wave along z since the dielectric slab is lossless. Therefore, the propagations constant k_z is real ($k_z = \beta_z$). This mode is characterized by a field that is exponential in the air and that is confined above the interface between the two media [17]. So, the wave number k_{y1} is real $k_{y1} = \beta_{y1}$ while β_{y1} is positive.By applying the difference between the two wave equations, we obtain the result equation:

$$k_{y2}^2 - \beta_{y1}^2 = -k_0^2(\varepsilon_r - 1) \quad (9)$$

Since $\varepsilon_r > 1$ and β_{y1} is a positive real, the wavenumber k_{y2} can be real or purely imaginary. In case k_{y2} is real, the dispersion equation can be written as:

$$\varepsilon_r \beta_{y1} = -\beta_{y2} th(\beta_{y2} d) \quad (10)$$

This equation implies that $\varepsilon_r < 0$ but it is not compatible with $\varepsilon_r > 1$. Therefore, the wave number k_{y2} is purely imaginary. At this condition, these surface modes can be obtained graphically by finding the intersection between two equations that are written as:

$$\begin{cases} \varepsilon_r \beta_{y1} = \alpha_{y2} \tan(\alpha_{y2} d) \\ \beta_{y1}^2 + \alpha_{y2}^2 = k_0^2(\varepsilon_r - 1) \end{cases} \quad (11)$$

The first equation corresponds to the tangent function. The second one corresponds to a circle with center (0,0) and radius a, where $a = k_0 d\sqrt{\varepsilon_r - 1}$. We will present in Fig.3.

2) TE Polarization

In the TE polarization, the electric fields can be written in the two regions as:

$$\begin{cases} E_{x1} = C_1 e^{-j k_z z} e^{-k_{y1} y} \\ E_{x2} = C_1 e^{-j k_z z} \sinh(k_{y2}(y + d)) \end{cases} \quad (12)$$

In this case, the dispersion equations can be written as:

$$\begin{cases} \beta_{y1} = -\alpha_{y2} \cot(\alpha_{y2} d) \\ \beta_{y1}^2 + \alpha_{y2}^2 = k_0^2(\varepsilon_r - 1) \end{cases} \quad (13)$$

As for the TM polarization, the intersection of these two equations allows us to find the surface modes which are supported in this structure. This intersection will be shown in Fig. 4.

B. *Volume Modes*

Volume modes propagate in the two media of this structure without radiation along the longitudinal z direction, and with a real propagation constant $k_z = \beta_z$. The magnetic field in the TM polarization is represented by these equations in the two regions:

$$\begin{cases} H_{x1} = C_1 e^{-j k_z z} e^{-j k_{y1} y} \\ H_{x2} = C_1 e^{-j k_z z} \cosh(k_{y2}(y + d)) \end{cases} \quad (14)$$

The difference between the two wave equations allows us to obtain this equation:

$$k_{y2}^2 + \beta_{y1}^2 = -k_0^2(\varepsilon_r - 1) \quad (15)$$

We imply that the wave number of the slab k_{y2} is purely imaginary. Therefore, we can write these two equations:

$$\begin{cases} \varepsilon_r \beta_{y1} = \alpha_{y2} \tan(\alpha_{y2} d) \\ \beta_{y1}^2 - \alpha_{y2}^2 = -k_0^2(\varepsilon_r - 1) \end{cases} \quad (16)$$

In order to find volume modes, we trace these two equations in the landmark ($\alpha_{y2} d/\pi, \beta_{y1} d/\pi$) for a height d=1.5cm and a relative permittivity ε_r.

In TE polarization, the electric field is written in the two regions by the two equations:

$$\begin{cases} E_{x1} = C_1 e^{-j k_z z} e^{-j k_{y1} y} \\ E_{x2} = C_1 e^{-j k_z z} \sinh(k_{y2}(y + d)) \end{cases} \quad (17)$$

Volume mode is characterized by a real wavenumber $k_{y1} = \beta_{y1}$ and a purely imaginary wavenumber $k_{y2} = -j\alpha_{y2}$. It is represented by the intersection of the two equations which are written as:

$$\begin{cases} \beta_{y1} = -\alpha_{y2} \cot(\alpha_{y2} d) \\ \beta_{y1}^2 - \alpha_{y2}^2 = -k_0^2(\varepsilon_r - 1) \end{cases} \quad (18)$$

The first equation corresponds to cotangent function. The second one corresponds to a circle with center (0,0) and radius a, where $a = k_0 d\sqrt{\varepsilon_r - 1}$.

C. *Leaky Modes*

The leaky mode propagates with radiation along the z direction [10]. It can radiate either forward or backward with a complex propagation constant $k_z = \beta_z - j\alpha_z$. The leaky mode is characterized by two complex wave numbers k_{y1} and k_{y2}.

In a conventional dielectric ($\varepsilon_r > 0$), the leaky mode is improper modes. But in a dielectric of a negative permittivity, the leaky mode is proper mode [4]. As a conclusion, in the studied structure only complex improper modes may exist, while complex proper modes are forbidden.

In case TE polarization, the dispersion equation can be written as:

$$W = -Z\coth(Z) \quad (19)$$

Where: $W = k_{y1}d = u - jv$ and $Z = k_{y2}d = x - jy$

By taking into consideration that $k_{y2}^2 + k_{y1}^2 = -k_0^2(\varepsilon_r - 1)$, we obtain the two equations:

$$\begin{cases}\cosh(2x)\cos(2y)(x^2 - y^2) - 2xy\sinh(2x)\sin(2y) = -a^2\\ \sinh(2x)\sin(2y)(y^2 - x^2) - 2xy\cosh(2x)\cos(2y) = 0\end{cases} \quad (20)$$

The solutions of this system illustrate the leaky mode for the grounded slab with relative permittivity $\varepsilon_r = 2.2$ and a height d=1.5cm.

IV. NUMERICAL RESULTS AND DISCUSSION

There are two kinds of surface modes, ordinary surface modes and surface plasmon [4]. According to our analysis, we conclude that only the trapped surface modes can exist in our structure because $k_0 < \beta_z < k_0\sqrt{\varepsilon_r}$. However, the existence of surface plasmon is conditioned by $\beta_z > k_0\sqrt{\varepsilon_r}$. It can be present in other structures like surface plasmon waveguides and plasma slabs. The ordinary surface mode is illustrated in Fig. 3 by a red cross, which represents the intersection of the tangent function and the circle whose center (0,0) and radius a for TM polarization.

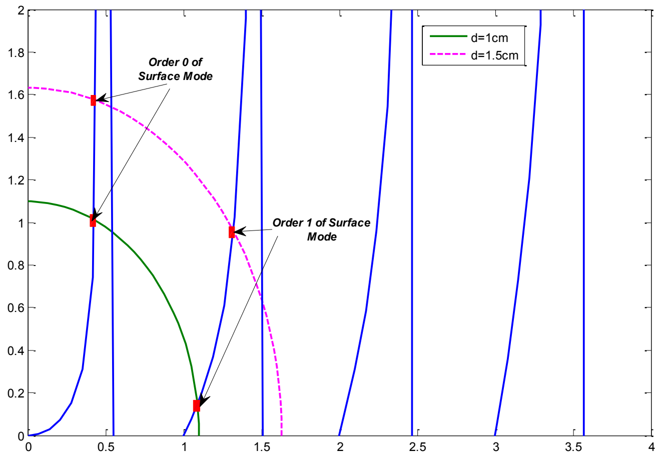

Figure 3. Location of TM surface modes for grounded slab with $\varepsilon_r = 2.2$. The green circle corresponds to d=1cm and the pink circle to d= 1.5cm.

For TE polarization, ordinary surface mode is illustrated in Fig. 4 by a red cross, which represents the intersection of the cotangent function and the circle whose center (0, 0) and radius a.

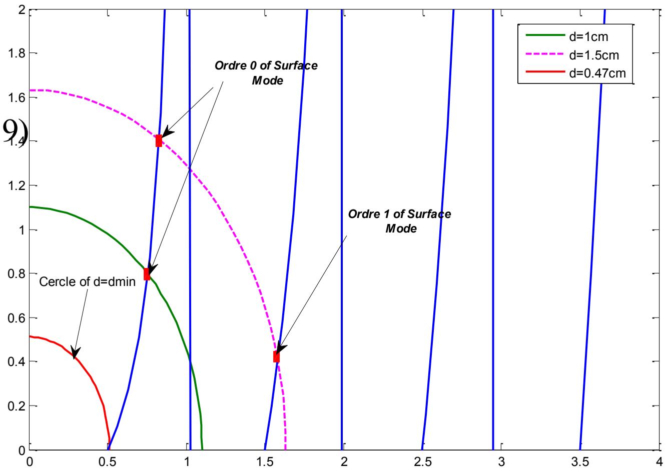

Figure 4. Location of TE surface modes for grounded slab with $\varepsilon_r = 2.2$ for three values of heigth d = 0.74cm (red circle),d=1cm (green circle) and d=1.5cm (pink circle).

When we increase the height d and the dielectric permittivity ε_r , the number of surface modes supported by this structure increases for both polarizations. This number is equal to n+1 with n as a natural number satisfying the relation $n - 1 \le a/\pi \le n$ [4].

In TE polarization, surface modes cannot exist below the limit value of radius a which is equal to 0.74cm. The red circle in Fig. 4 represents the limiting case.

Fig. 5 shows the dispersion characteristics of the surface mode for TM and TE polarizations supported by a grounded slab with relative permittivity $\varepsilon_r = 2.2$, permeability $\mu_r = \mu_0 = 1$ and height $d = 1.5cm$.

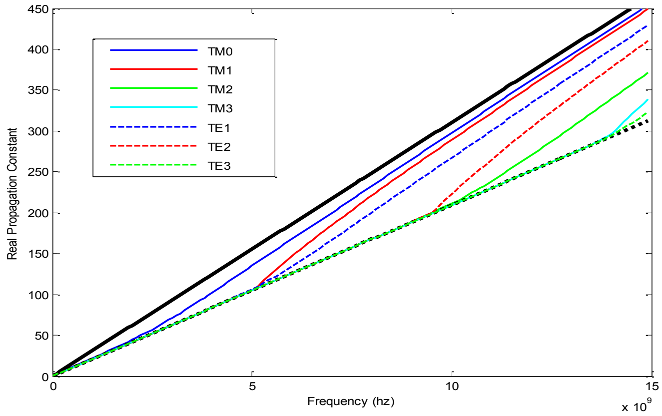

Figure 5. Dispersion plots of TM and TE ordinary surface modes for grounded slab showing the real propagation constant (β) versus the operating frequency (f) . The dielectric slab is characterized by $\varepsilon_r = 2.2$, $\mu_r = \mu_0 = 1$ and d = 1.5cm.

One of the dispersion characteristics of grounded slab is that the cutoff of the principal mode TM $_0$ of surface mode does exist, as shown in Fig. 5. But, in case $\mu_r \neq 1$, the TM $_0$surface modes cutoff is suppressed.
The cutoff frequency of each guided modes of higher-order is also shown in Fig. 5 for TM and TE polarizations. The cutoff frequency is the lowest frequency of guided propagation at which $\beta_z = K_0$ [18]. For higher-order modes, when the frequency is increased, the propagation constant approaches to $K_0\sqrt{\varepsilon_r}$ but it cannot exceed this value ($K_0\sqrt{\varepsilon_r}$). This limit is the upper limit of ordinary surface modes, whereas, the lower limit is k_0. This means that the ordinary surface modes are slow. When the relative permittivity of slab is increased, the propagation constant decreases and the lost power is confined near the slab region.

In Fig. 6 and Fig. 7, the graphic location of the second kind of real modes, i.e., volume mode are represented in TM and TE. Volume mode is represented by a red cross which is the intersection of two curves. The first curve represents the tangent function for TM polarization and cotangent for TE polarization while the second curve represents the hyperbola function.

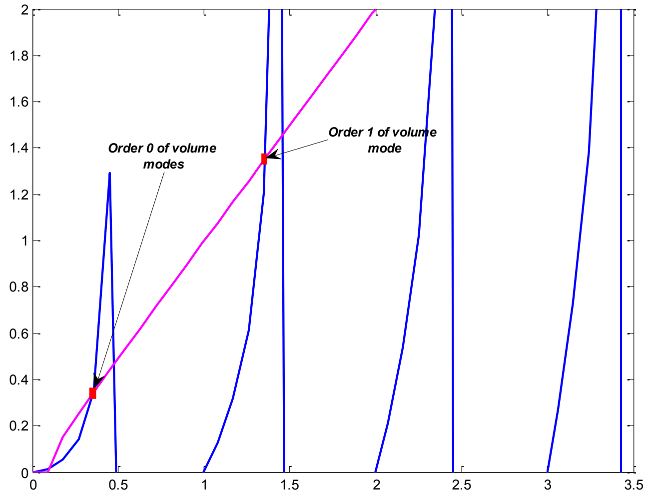

Figure 6. Location of TM volume modes for grounded slab of height d=1.5cm with $\varepsilon_r = 2.2$ and $\mu_r = \mu_0 = 1$.

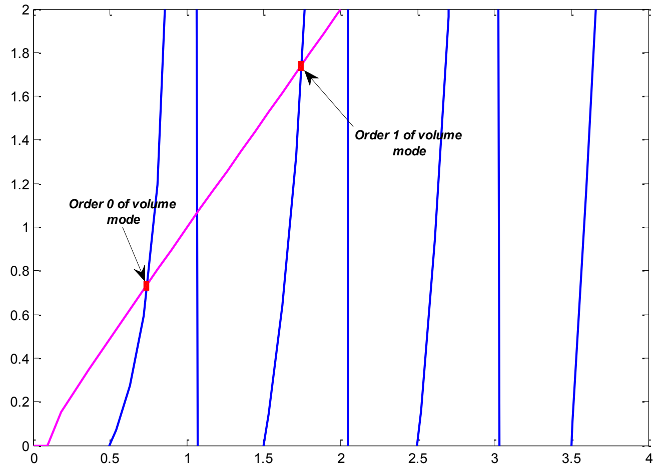

Figure 7. Location of TE volume modes for grounded slab of height d=1.5cm with $\varepsilon_r = 2.2$ and $\mu_r = \mu_0 = 1$.

We can see that when we increase the height d of the structure, the number of the volume mode decreases for the two polarizations, and the principal mode TM_0 can be suppressed. This number equal to m-1, when m is a natural number satisfying the relation $m-1 \leq a/\pi \leq m$.

In Fig. 8, the dispersion characteristics of volume mode (for the real propagation constant versus frequency) are represented in TM and TE polarizations for the grounded slab structure with $\varepsilon_r = 2.2$ and $\mu_r = \mu_0 = 1$.

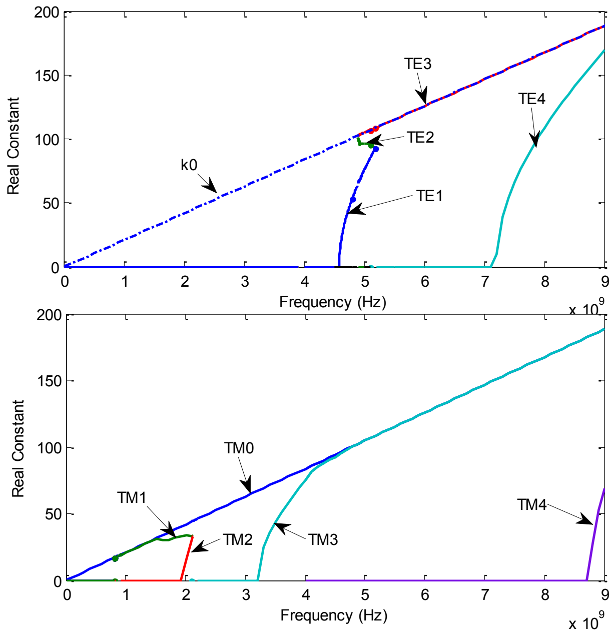

Figure 8. Dispersion plots of TE and TM volume modes for grounded slab of heigth d = 1.5cm with a relative permitivity $\varepsilon_r = 2.2$.

Through Fig. 8, we observe that the real propagation constant cannot exceed the propagation constant in free space (i.e., $\beta_z > K_0$). Thus, the volume mode for both polarizations exists in the grounded slab structure. The representation of Fig. 8 shows that the dispersion curve of first mode of higher-order TM_1 tends, on the one hand, towards the fundamental mode TM_0 in low frequencies, and disappears, on the other hand, when the frequency $f = 2.1GHz$. The dispersion curve of the third mode of higher-order TM_3 tends toward that of the principal mode TM_0 of volume mode in high frequencies. In polarization TE, there is no fundamental mode. The mode of higher-order TE_2 appears at the same frequency of TE_3 where f= 4.89GHz and disappears at the same frequency of TE_1 where f=5.18GHz.

In grounded slab with positive permittivity and permeability, just one type of the complex mode exists, which is the improper mode [7]. The dispersion characteristics of these modes are represented in Fig. 9. The dielectric slab is characterized by height d=1.5cm, $\varepsilon_r = 2.2$ $\mu_r = \mu_0 = 1$ and height d = 1.5cm.

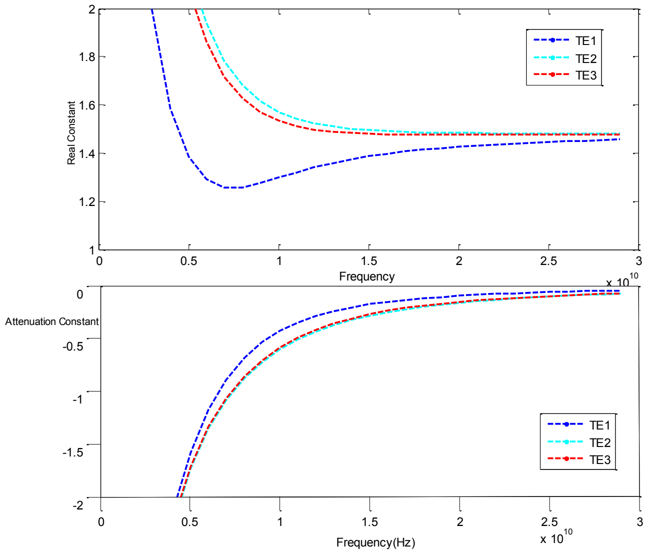

Figure 9. Dispersion curves of normalized real and attenuation constants of TE leaky modes for grounded dielectric slab .

We can see that leaky modes exist in the grounded slab when $\beta_z > K_0$. When the frequency is lower than the first cutoff frequency of the surface mode $f_{c1} = 4.4GHz$, all complex modes exist with very high normalized β_z and α_z. When the frequency increases, the normalized real constants $(0,0)$ decrease rapidly within a very narrow frequency range and remain constant at $\sqrt{\varepsilon_r}$. However, the normalized attenuation constants α_z / k_0 increase rapidly and remain constant at zero. Leaky mode is characterized by an order mode n. The minimum of n for leaky mode must be equal to the maximum of this order mode for surface mode [4].

V. CONCLUSIONS

In this paper, we investigate the dispersion characteristics and existence conditions of real and complex TE and TM modes. These modes are separated by the nature of the propagation constant. Surface mode is characterized by $k_0 < \beta_z < k_0\sqrt{\varepsilon_r}$, volume mode is characterized by $\beta_z < k_0 < k_0\sqrt{\varepsilon_r}$ and leaky mode is characterized by $\beta_z > K_0$. We observe that only the improper type of the leaky mode and only ordinary type of the surface mode occur in the double-positive grounded slab. The analysis are achieved by a simple method which is the TRM.

REFERENCES

[1] A. Baussard and S. Grosdidier, 'D'etection de cibles par radar HFSW :utilisation des curvelets et des ondelettes continues', Laboratoire EI-ENSIETA.

[2] V. Laude, 'Acoustique physique-physical acoustics', Institut FEMTO-ST.

[3] J. Picaud, 'Biodétection par résonance des plasmons de surface localisés', decembre 2009.

[4] O. Balosso,'Introduction des Ondes de surface dans un Modèle Adapté de Faisceaux Gaussiens pour le Traitement du Couplage Antenne-Structure', thèse, université Toulouse 3 Paul Sabatier pp. 1-41, 2014.

[5] A. Alu, F. Bilotti, N. Engheta and L. Vegni, 'Subwavelength Planar Leaky-Wave Components With Metamaterial Bilayers',IEEE Trans. Antennas Propag., vol. 55, no. 3,March 2007.

[6] K. Y. Kim, Young Ki Cho and Heung-Sik Tae,'Guided Mode Propagations of Grounded Double-Positive and Double-Negative Metamaterial Slabs with Arbitary Material Indexes', May 2006.

[7] P. Baccarelli, P. Burghignoli, F. Frezza, A. Galli , P. Lampariello, G. Lovat and S. Paulotto,'The nature of radiation from leaky waves on single-and double-negative metamaterial grounded slabs', IEEE MTT-S Digest, 2004..

[8] P. Baccarelli, P. Burghignoli, G. Lovat and S. Paulotto, 'Surface-wave Suppression in a Double-Negative Metamaterial Grounded Slab', 2003.

[9] V. H. Mok and L. E. Davis, 'Electromagnetic Surface Waves in Mixed Media', Munich 2003.

[10] P. Baccarelli, P. Burghignoli, F. Frezza, A. Galli, P. Lampariello, G. Lovat and S. Paulotto, 'Effects of Leaky-Wave Propagation in Metamaterial Grounded Slabs Excited by a Dipole Source', IEEE Trans. Micro. Teoey Tech, vol. 53, no. 1, January 2005.

[11] H. Aubert, H. Baudrand, 'L'electromagnetisme par les schemas équivalents', pp. 22-p34.

[12] N. Raveu, O. Pigaglio, 'Résolution de problémes hautes fréquences par les schémas équivalents', pp. 32-48

[13] S. B. Fonseca and A. J. Giarola, 'Surface wave excitation of microstrip disk antennas', IEEE Trans. Antennas Propag, vol 12, no. 10, pp. 426-429, 1984.

[14] D. G. Duffy, 'Green's Function with Applications', Boca Raton: Chapman and Hall/CRC, 2001.

[15] S. J. Orfanidis, 'Electromagnetic Waves and Antennas', ch. 7, pp. 238-272, Piscataway: E-Book, 2008

[16] T. Tamir and A. A. Oliner, 'Guided complex Waves', Prog. IEEE, vol. 110, no. 2, pp.310-334, Feb 1963.

[17] D. M. Pozar,'Microwave Engineering', New York: John Wiley and Sons, 2004.

[18] K. Y. Kim, Young Ki Cho and Heung-Sik Tae,'Guided Mode Propagations of Grounded Double-Positive and Double-Negative Metamaterial Slabs with Arbitary Material Indexes', Journal of the Korean Physical Society, vol. 49, no. 2, August 2006, pp. 577-587.