Intercomparisons of Inertial Heading Sensors: Reference Sensor with Zero Systematic Error

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***Abstract*—The paper formulates the problem of intercomparing inertial heading sensors to check if the tested sensor complies with the specified requirements. We show in this paper that the problem can be solved if the error of the reference sensor is known with sufficient accuracy. This study creates perspectives for solving the problem of calibration against the primary standard during marine tests of heading sensors.**

Keywords-sensor; heading sensor; inertial sensor; sensors intercomparison; sensor error; systematic error; random error

I. Introduction

Intercomparisons of inertial heading sensors are generally conducted to determine their corrections during pre-cruise preparation and to check their serviceability during operation [1]–[3]. These intercomparisons have two important features: first, more accurate sensors can be used for correction determination, and second, a small volume of data samples is available due to limited time of intercomparison.

Accuracy characteristics of the heading sensor during marine tests are traditionally determined by the same methods because we are supposed not to have heading standard at sea. By the latter we mean the device determining heading with an error negligibly small compared with the error of the tested sensor. Indeed, achieving a constant heading and keeping it does not solve the problem as the heading keeping accuracy is a fortiori lower than the required measurement accuracy (for successful intercomparison). Using Global Positioning System (GPS) data (if available) for heading calibration is usually impossible due to insufficient accuracy (with required update rate) or insufficient data rate (with required accuracy). Insufficient accuracy is even more degraded by the ship deformations, which make it impossible to install GPS receivers at the desired distance.

Thus, the heading sensor error is determined as follows during the tests: in static mode, the geodetic direction known to the desired accuracy is transferred to the ship, and the deviation between this direction and the direction of the sensor such as the axis of inertial trihedral formed by the heading sensor is determined. For higher reliability, the procedure is repeated before the ship goes to sea for marine tests and after its return. Clearly, in this case the heading sensor is not tested in the most complicated dynamic heading generation mode, during maneuvering in the open sea. At the same time, there are no obstructions for using a traditional metrological procedure of *intercomparison of measures* [4] to determine the heading accuracy.

This paper aims to demonstrate the possibility of checking the *tested heading sensor* (TS) by intercomparing it with another sensor further referred to as a *reference* (RS). In this paper, we assume that the RS generates heading with an error known to the required accuracy and without any systematic error. It is the set of two parameters – the ratio between the errors of the TS and RS, and the accuracy of estimating the RS error – which defines the test success. In the rest of the article we will refer of estimating both the systematic and random components of the TS error.

The rest of the paper is structured as follows. In Section II, we formulate the problem and, in Section III, we present the solution to the formal problem. The paper concludes in Section IV.

II. Problem Formulation

To reach this aim, the paper discusses the variants of the general problem of processing heading data generated by the intercompared sensors, which is formulated as follows.

As the heading sensors are installed onboard the ship, the physical (measured) heading axes of the TS and RS are aligned with the ship centerline plane accurate to Φ and Φ_{ref}, respectively. Thus, the relative offset of the axes is $\Delta\Phi = \Phi - \Phi_{ref}$ (see Fig. 1). Because of that, RS has zero systematic error, and we can ensure $\Phi_{ref} = 0$ by the special measuring procedure. So, it is supposed further that $\Delta\Phi = \Phi$, and, besides, Φ is significantly less than the error norm of TS. The general error model used in the paper is a random value with expected value (at start) presents a systematic error of a TS (denoting below by *a*). So we consider intercomparisons of a TS and RS where the latter has zero systematic error. As a rule, heading sensors error is characterized by *standard deviation* (denoting below by σ and σ_{ref}) or confidence limits. This paper mostly addresses normalization of standard deviation of random error (setting σ_{max} and $\sigma_{ref,\,max}$), and setting confidence limits is considered to be the secondary normalization method. During the tests,

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the sensors output current heading arrays $\{\varphi_i\}$ and $\{\varphi_{\mathrm{ref},\, i}\}$ at discrete time moments $\{t_i\}$, where $i = 1, \ldots, n$. The array volume n is determined by the test duration and sampling rate of digital signals (data readout rate). Usually, several runs are made during the tests, each being several hours long (up to 10–12 hours). The initial period of each run is occupied by the device thermal stabilization (and other kinds of stabilization). Thus, with sampling rate of the order of (1–100) Hz the volume of array n can reach several thousands and more.

It is supposed that the array data are generated synchronously, which makes it possible to generate the difference array $\{x_i\}$, where $x_i = \varphi_i - \varphi_{\mathrm{ref},\, i}$. Definitely, readings $\{\varphi_{\mathrm{ref},\, i}\}$ include information on the accuracy of RS, though in latent form, since true heading $\varphi_0 = \varphi_0(t)$ at each time t is unknown.

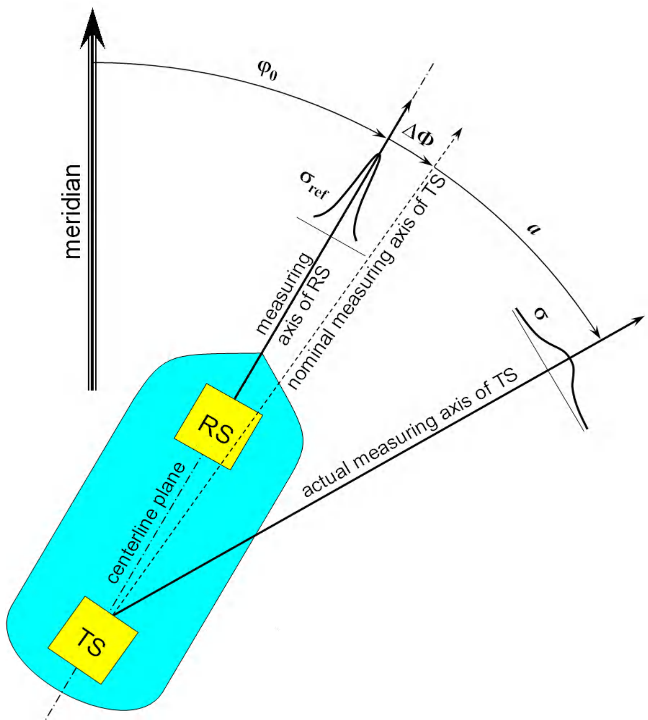

Figure 1. Measuring scheme of intercomparisons TS and RS.

Even if the design of RS provides zero heading systematic error, the ratio of spectra of sensor random error and true heading as a function of time generally remains unknown, which does not allow setting adequate averaging parameters for random error to find the true heading. Then, for a good RS, true σ_{ref} may turn out to be less than $\sigma_{\mathrm{ref,\, max}}$, however, since it is unknown, statistical conclusions about the TS based on the normative parameters of RS will be less reliable. True rather than normative parameters can be obtained using two methods. First, true parameters are determined during the installation of RS onboard the ship and after its repair, and are recorded in the sensor service log. Second, if the spectrum of true heading can be determined, one can try to estimate the variance of its random error component by filtering the low-frequency part. Keeping these possibilities in mind, further we suppose the normative parameter $\sigma_{\mathrm{ref,\, max}}$ to be rather close to the true value of σ_{ref}.

Clearly, the difference of readings x can be represented by symbolic sum of initial installation error $\Delta\Phi$ and the errors of intercompared sensors ξ, ξ_{ref}:

$$x = \Delta\Phi \oplus \xi \oplus \xi_{\mathrm{ref}}. \qquad (1)$$

Equation (1) is the principal methodological basis of the test method by intercomparison TS with RS.

Further the array $\{x_i\}$ is considered to be a sample the of parent population generated by the sum of quasideterministic drift and ergodic stationary process modeling the sensor inaccuracy in steady mode. From the experience of development and operation of heading sensors, the drift nature of a certain device is assumed not to change from run to run. When modeling the drift by an exponential function, it means that the number of elements (power) of polynomial remains unchanged, and only its coefficients change. As to the stochastic part of this sum, assumption on its character requires thorough experimental check. The assumption is based on the fact that development and check-out of the device is a long multistep process aimed at stabilization of sensor construction in real operating conditions. As to the hypothesis of stationary stochastic part of the above mentioned sum, it relies on the assumption of sensor accuracy independency on the current heading (and its variations). This assumption, and therefore, stationarity hypothesis, requires verification. This verification is quite possible while the samples are large. The ergodicity hypothesis can be checked only partially (check of equality of ensemble and time averages) because of limited number of runs. At the same time, it should be emphasized that error time series can be treated as samples of the relevant parent populations on ergodicity assumption only.

Assuming that the data of the intercompared sensors received in each run after stabilization are analyzed, the elements of array $\{x_i\}$ can be considered to be the differences of random values ξ_i and $\xi_{\mathrm{ref},\, i}$ characterized by mathematical expectations a, a_{ref} and variances σ^2, σ_{ref}^2 (or standard deviations σ, σ_{ref}). It is supposed that the moments of each value ξ_i, $\xi_{\mathrm{ref},\, i}$ can change from run to run but remain unchanged within a run. Then due to large arrays of samples, sampling moments will practically coincide with the relevant probability characteristics, i.e., the moments of the considered parent populations.

We are not supposed to have information on spectral characteristics of errors of the intercompared sensors, firstly, the RS, thus, the ship true heading φ_0 cannot be determined by filtering the sensor readings.

Accuracy requirements on the TS are set as norms of statistical characteristics of its error. First of all, standard deviations σ and confidence limits ($a \pm K(P)\cdot\sigma$; P, F) are used, where P is the given confidence probability, F is the accepted probability distribution law. In practice, P=0.997 is often used, which corresponds to the confidence limits equal to the tripled standard deviation σ for symmetrical Gaussian distribution. In this case the problem of processing of data received by intercomparing the heading sensors is formulated as follows:

(*a*) by analyzing the array $\{x_i\}$, $i = 1, \dots, n$ of difference of output signals φ_i and $\varphi_{\text{ref},\,i}$ from the intercompared sensors determine, according to the substantiated criterion, whether actual standard deviation σ of the error of TS (or confidence limit) agrees with the required norm $\sigma_{\max}$ for the given data array (run);

(*b*) repeat (*a*) for all arrays (runs) and determine the maximum (worst) estimates of these parameters.

Therefore, further research is aimed to develop solution methods for the above given formal problem and its components.

This formal problem falls into several variants depending on a priori information on parameters of sensor data arrays. Here, we restrict ourselves to a practically significant case of a RS *without systematic component* in the error. The variants are presented in Table 1. Notation $a\in[a_{\text{low}}, a_{\text{up}}]$ means that sample average changes from run to run but remains unchanged within a run. Notation a = const means that the sample average remains unchanged also from run to run.

TABLE I. VARIANTS OF PROBLEM CONDITIONS

Variant	A priori information on parameters of		Analyzed parameters of		Note
	array $\{\varphi_{\text{ref},\,i}\}$	array $\{\varphi_i\}$	RS	TS	
A	$a_{\text{ref}} = 0$	$a = 0$	σ_{ref}	σ	–
B	$a_{\text{ref}} = 0$	a = const	σ_{ref}	a, σ	–
C	$a_{\text{ref}} = 0$	$a\in[a_{\text{low}}, a_{\text{up}}]$	σ_{ref}	$a(j)$, σ	$j = 1, \dots, m$ – run no.

Consider three combinations of parameters given in Table 1 step by the step using the same criteria.

III. SOLUTION OF THE FORMAL PROBLEM

A. Random distortions of recorded signals only: $a_{\text{ref}} = a = 0$

A1. To receive analytical solutions, with account for experience in studying the heading sensors, the character and volume of test data, and with account for the fact that any sampling moment is asymptotically normal [5], it would be expedient to use the following stochastic model with unbiased normal (Gaussian) distribution *N* of both sensor errors as a basic model:

$$\begin{cases}\varphi_{\text{ref}}(t) = \varphi_0(t)+\xi_{\text{ref}}(t),\ \ \xi_{\text{ref}}(t) \sim N(0, \sigma_{\text{ref}}^2) \text{ with any } t=t_i,\\ \varphi(t) = \varphi_0(t)+\xi(t),\ \ \xi(t) \sim N(0, \sigma^2) \text{ with any } t=t_i,\end{cases} \quad (2)$$

where σ_{ref} is the standard deviation of random error of RS (known); σ is the standard deviation of random error of TS (to be determined); $\varphi_0(t)$ is the unknown true heading at time *t*. The readings are supposed to be mutually noncorrelated: $\text{cov}[\xi(t), \xi_{\text{ref}}(t)]=0$ with any $t=t_i$.

Under these conditions, the following is true for the variance $D_x = D[x(t)]$ of difference $x(t) = \varphi(t) - \varphi_{\text{ref}}(t)$ of readings of two intercompared sensors:

$$D_x = D\left[\xi(t) - \xi_{ref}(t)\right] = \sigma^2 + \sigma_{ref}^2. \quad (3)$$

Expression (3) serves as a basis for the criterion to check whether true standard deviation of the TS error complies with the requirements. Determine the sample variance $\widetilde{D}_x$ of differences $x_i = \varphi_i - \varphi_{\text{ref},\,i}$ of sensor discrete readings:

$$\widetilde{D}_x = \sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1) \quad (4)$$

where $\bar{x} = n^{-1}\cdot\Sigma x_i$, *n* is the sample size, and with account for its proximity to the parent population variance (3), compare it with the sum of norms for error variances taken as an acceptable upper estimate (4). If the given normative limit

$$\widetilde{D}_x > \sigma_{\max}^2 + \sigma_{ref,\max}^2 \quad (5')$$

is exceeded, where $\sigma_{\max}$, $\sigma_{\text{ref, max}}$ are the norms of standard deviation of the TS and RS, the TS is known *bad*, while then definitely $\sigma > \sigma_{\max}$. However, to confirm the *fitness* of the TS (so that the inequality $\sigma \le \sigma_{\max}$ is undeniably valid), this limit should be toughened while checking

$$\widetilde{D}_x \le \sigma_{\max}^2 + M^2\cdot\sigma_{ref,\max}^2, \quad (5'')$$

where coefficient $M = \inf\{\sigma_{\text{ref}}/\sigma_{\text{ref, max}}\} \le 1$ determines the criterion "dead zone":

$$M < \frac{\sqrt{\widetilde{D}_x - \sigma_{\max}^2}}{\sigma_{ref,\max}} \le 1. \quad (5''')$$

This zone has a relative width 1–*M* and characterizes the acceptable risks of the manufacturer and the customer.

It should be also noted that criterion (5) is invariant to the type of error distribution, and in this sense is far beyond the framework of model (2). However, it ignores the possible random error of total estimated variance (4), and thus has a deterministic nature.

A more detailed criterion of sensor fitness can be based on setting the confidence limit for the module of error difference $|\Delta\xi|$ equal to the module of difference of readings from the intercompared sensors: $|\Delta\xi_i| = |\xi_i - \xi_{\text{ref},\,i}| = |\varphi_i - \varphi_{\text{ref},\,i}| = |x_i|$. While differences $\{x_i\}$ retain their Gaussian distribution in conditions of model (2), the confidence limit for $|\Delta\xi|$ is $\Delta_{conf} = 3\sqrt{\widetilde{D}_x}$ (P=0.997), where $\widetilde{D}_x$ is calculated by (4). Thus, in accordance with this criterion, the TS is considered *bad* if condition

$$n^{+} > n(1-P) \quad (6')$$

holds, where n^{+} is the number of "extreme" differences of samples x_i exceeding the acceptable limits

$$\pm 3\sqrt{\sigma_{\max}^2 + \sigma_{ref,\max}^2}. \quad (*)$$

Then, to rank the TS as *good* we should count the number $n^{++} > n^{+}$ of "extreme" differences of samples x_i exceeding the toughened limits

$$\pm 3\sqrt{\sigma_{\max}^2 + M^2\cdot\sigma_{ref,\max}^2}, \quad (**)$$

where *M* is determined in (5″), and check if condition

$$n^{++} \le n(1-P) \quad (6'')$$

is fulfilled.

It can easily be seen that the dead zone of this criterion is given by

$$n^{+} \le n(1-P) < n^{++} \qquad (6''')$$

and has a relative width $(n^{++} - n^{+})/[n(1-P)]$. Note that criterion (6) is highly sensitive to anomalous outliers in raw data.

A2. If it cannot be established that the TS error has a Gaussian distribution, and some interval $[-\Delta, \Delta]$ exists only which embraces its values, the following model should be used instead of model (2):

$$\begin{cases} \varphi_{\mathrm{ref}}(t) = \varphi_0(t)+\xi_{\mathrm{ref}}(t), \quad \xi_{\mathrm{ref}}(t) \sim N(0, \sigma_{\mathrm{ref}}^2) \text{ with any } t=t_i, \\ \varphi(t) = \varphi_0(t)+\xi(t), \quad \xi(t) \sim U(-\Delta, \Delta) \text{ with any } t=t_i, \end{cases} \qquad (2a)$$

where $U(-\Delta, \Delta)$ is the uniform distribution of TS errors, Δ is the limiting error module (which is to be estimated and should not exceed the given normative error $\Delta_{\max}$). This is justified because the error is set by its limits and uniform distribution is the worst variant of unimodal distributions [6]. Here, for the sake of uniformity of statistical methods in use, instead of Δ further we will consider equivalent standard deviation of uniform distribution $\sigma = \Delta/\sqrt{3}$ with the norm $\sigma_{\max} = \Delta_{\max} / \sqrt{3}$.

In this case, expressions (3)–(6) remain in force with relevant changes of coefficient K=3 for the standard deviations in (*), (**) through composition $N{*}U$ of distributions in model (2a) [5], [6].

A3. If the limits of RS error are specified, similarly to (2a) we have

$$\begin{cases} \varphi_{\mathrm{ref}}(t) = \varphi_0(t)+\xi_{\mathrm{ref}}(t), \ \xi_{\mathrm{ref}}(t) \sim U(-\Delta_{\mathrm{ref}}, \Delta_{\mathrm{ref}}) \text{ with any } t=t_i, \\ \varphi(t) = \varphi_0(t)+\xi(t), \quad \xi(t) \sim N(0, \sigma^2) \text{ with any } t=t_i, \end{cases} \qquad (2b)$$

where $\sigma_{\mathrm{ref}} = \Delta_{\mathrm{ref}}/\sqrt{3}$ is known; σ is to be estimated. For this case, as well as above, modified formulas for the criteria (5)–(6) can be received through composition $N{*}U$.

A4. If there are no grounds to present the errors of both sensors using Gaussian distribution, uniform distribution should be used similar to models (2a) and (2b):

$$\begin{cases} \varphi_{\mathrm{ref}}(t) = \varphi_0(t)+\xi_{\mathrm{ref}}(t), \ \xi_{\mathrm{ref}}(t) \sim U(-\Delta_{\mathrm{ref}}, \Delta_{\mathrm{ref}}) \text{ with any } t=t_i, \\ \varphi(t) = \varphi_0(t)+\xi(t), \quad \xi(t) \sim U(-\Delta, \Delta) \text{ with any } t=t_i, \end{cases} \qquad (2c)$$

where $\sigma_{\mathrm{ref}} = \Delta_{\mathrm{ref}}/\sqrt{3}$ is known; $\sigma = \Delta/\sqrt{3}$ is to be estimated. Here, modified formulas of criteria (5)–(6) can also be received through composition $U{*}U$.

Thus, in the considered cases, where the assumption on Gaussian distribution of random errors of intercompared sensors is not fulfilled, the solution to the formulated problem exists and can be received using formulas (3)–(6) or their simple modifications.

B. RS has random distortions only: $a_{ref} = 0$, and TS has both random and constant systematic signal distortions: $a = const$

By distortions, we mean inaccurate initial zero setting and constant error in certain runs of the TS, and if the data received in several runs are processed, a is considered to be constant from run to run. Thus, the following biased stochastic model with Gaussian error distribution is taken:

$$\begin{cases} \varphi_{\mathrm{ref}}(t) = \varphi_0(t)+\xi_{\mathrm{ref}}(t), \ \xi_{\mathrm{ref}}(t) \sim N(0,\sigma_{\mathrm{ref}}^2) \text{ with any } t=t_i, \\ \varphi(t) = \varphi_0(t)+\xi(t), \quad \xi(t) \sim N(a,\sigma^2) \text{ with any } t=t_i, \end{cases} \qquad (2d)$$

where σ_{ref} is known, $\{a, \sigma\}$ are to be estimated on the assumption that the readings of intercompared sensors are noncorrelated: $\mathrm{cov}[\xi_{\mathrm{ref}}(t), \xi(t)]=0$ with any $t=t_i$. Then mathematical expectation a is not known, though its estimation is needed merely to estimate the standard deviation σ.

Obviously, in these assumptions for the variances the relation (3) is still valid. Then, while the bias a is constant, parameter D_x can be estimated by the same formula (4). It should be noted that criteria (5)–(6) can be modified with the account for these particular conditions.

Getting back to formula (4), note that the expression for $\bar{x}$ used in it is the estimate of the bias a of the TS. Thus, the confidence limits of its total error in this case are

$$\begin{cases} \Delta_{conf,\,low} = \bar{x} - K\sqrt{\widetilde{D}_x - \sigma_{ref}^2} \\ \Delta_{conf,\,up} = \bar{x} + K\sqrt{\widetilde{D}_x - \sigma_{ref}^2} \end{cases} \qquad (7)$$

with $K=K(P)=3$ for P=0.997 (in Gaussian case).

We can also apply standard methods to check the hypothesis of normal distribution of pairwise differences of samples $\{\varphi_{\mathrm{ref},\,i}\}$, $\{\varphi_i\}$, and with some rather general conditions for unknown signal $\varphi_0(t)$, of the samples themselves (after reasonable correction). For example, transformation to symmetrical first differences of each sample $\delta\varphi_{\mathrm{ref},\,i} = \varphi_{\mathrm{ref},\,i-1} - \varphi_{\mathrm{ref},\,i+1}$ and $\delta\varphi_i = \varphi_{i-1} - \varphi_{i+1}$ actually compensates the contribution of alternating signal $\varphi_0(t)$. Then their normal distribution (if such) will be maintained. The other method to suppress $\varphi_0(t)$ consists in calculating the symmetrical moving average (with a window of appropriate width) for each realization with further subtraction of the result. Samples corrected by this method (i.e., reduced to "zero" input signal) should be checked for normality.

Other than Gaussian error distributions can be considered similar to variants (*A2*)–(*A4*), and with relevant changes of coefficient K=3 for the standard deviations in (7).

C. RS has random distortions only: $a_{ref} = 0$, and TS has both random and constant systematic signal distortions, with the latter changing from run to run: $a \in [a_{low}, a_{up}]$.

If only the standard deviation σ is to be estimated, variant (*B*) should be repeated m times (with different a) and take the worst estimates by (4)–(6). If we are interested in total error, we search for confidence limits of the form $a \pm K(P)\cdot\sigma$ (also the worst among m runs), where $K(P)$ corresponds to the composition of the normal N and uniform U distributions (in various combinations $N*N$, $N*U$, $U*U$).

IV. Conclusions

Analysis and formulation of the problem of intercomparing an inertial heading sensor with some reference sensor to check the accuracy of the former show that the problem can be solved. Compliance or noncompliance of the tested sensor error to the specified norms can be reliably established. Traditional condition of sensors intercomparing has been analyzed and extended. We proceeded on the following assumptions: (a) the volume of an initial data (the sample size) under processing is large ($n \gg 1$); (b) the array data from the sensors are generated synchronously; (c) the normative parameter $\sigma_{\text{ref, max}}$ is rather close to true value of σ_{ref}; (d) the difference array $\{x_i\}$ is a sample of parent population generated by the sum of quasideterministic drift and ergodic stationary process; (e) the drift nature of a certain device is assumed not to change from run to run; (f) the moments of each value ξ_i, $\xi_{\text{ref}, i}$ can change from run to run but remain unchanged within a run; (g) the mathematical expectation of the reference sensor data equals zero; (h) the readings are supposed to be mutually noncorrelated: $\text{cov}[\xi(t), \xi_{\text{ref}}(t)]=0$ with any $t=t_i$; (j) all pairs of the popular stochastic models (Gaussian and uniform distributions) for the sensor data are considered.

It is shown that the problem can be solved in the presence of accurate estimate of characteristic of reference sensor error, particularly, the estimate close to the established accuracy norm. The obtained theoretical relationships and dependencies can serve as a basis for developing procedures for heading sensor intercomparisons, and under the defined conditions, solve the problem of calibration against the primary standard during marine tests of the heading sensors.

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