Rotated Constellations with Scaled Factor for High-Rate Full-Diversity STBC of 2 and 4 Antennas

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***Abstract*—We design a new $rate-5/4$ full-diversity orthogonal space-time block code (STBC) transmission scheme for QPSK with 2 transmitting antennas (TX) by one modified QPSK constellation with rotated and scaled factor by maximizing the CGD (coding gain distance) from the set of quaternions used in the Alamouti code. A low-complexity maximum likelihood (ML) decoding algorithm has been proposed that not only provide good FER (frame error rate) as same as $rate-1$ but also increase the transmitted rate approach the $rate-9/8$ without additional bandwidth. Finally, we extend the design to the case for 4 TX with low complexity by enlarging the set of Quasi-Orthogonal STBC proposed in without power scaling. Extension to general M-PSK constellation is also straightforward is the simulation results. The simulation result shows that the proposed scheme provides a better Channel Throughput performance than the original work.**

Keywords-STBC; ML; Constellation; High-rate; MPSK.

I. INTRODUCTION

SPACE–TIME trellis codes have been introduced in [1] to provide an improved error performance for wireless systems with multiple transmit antennas. The authors have shown that such codes can provide full diversity gain as well as additional signal-to-noise ratio (SNR) advantage that they call the coding gain. Code design rules for achieving full diversity are also provided. Using these design rules, examples of codes with full diversity as well as some coding gain constructed are not necessarily optimal. Since there is no general rule for designing codes that provide diversity as well as coding gain, it is unclear how to design new codes for different number of states or different rates. Also, it is not clear how to improve the performance of the codes, i.e., how to maximize the coding gain. There have been many efforts to improve the performance of the original space–time trellis codes [2]–[5]. While very interesting codes have been proposed in the literature, the coding gain improvements are marginal for one receive antenna.

In [8], the authors have proposed a class of STBCs for a high rate transmission scheme by exploiting the inherent algebraic structure for 2 and 4 transmit antennas. Compared with Alamouti and Jafarkhani schemes, the diversity gain is attenuated due to rotating or scaling one transmit matrix selected from the class.

In this work, we provide a new structure for space–time trellis codes that guarantees full diversity and provides opportunity to maximize the coding gain. We also provide a systematic method to maximize the coding gain for a given rate (rate>1), constellation, and number of states. The main idea of the proposed scheme is to employ two different signal constellations, and each constellation contains the information of the transmitted bit. In order to achieve the best system performance, the design criterion is also introduced in this paper to maximize the CGD to get the optimum rotated angle and scaled factor. The simulation is also given to demonstrate the reliability of the proposed transmission scheme.

The organization of the paper is as follows. In Section II, we briefly described the transmission scheme. Then, the proposed constellation is briefly introduced in Section III. In Section IV, the proposed algorithm is also produced to improve the system performance. Finally, simulation results are used to demonstrate the proposed scheme in Section V.

II. TRANSMISSION SCHEME

The objective in this paper is to design a new transmission scheme for high-rate (> 1) space-time block codes (STBC) by exploiting the QPSK constellation structure in existing orthogonal designs based on quaternions for 2 transmitting antennas [1] and quasi-orthogonal designs for 4 transmitting antennas [5]. The simplest example of a complex orthogonal design is the 2×2 code

$$G(x_1, x_2) = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \tag{1}$$

proposed by Alamouti [1], where $(\cdot)^*$ denotes the complex conjugate transpose and $x_i \in s, i = 1,2$ and $s \in \left\{1/\sqrt{2}\cdot(\pm 1 \pm i)\right\}$. This code achieves rate-1 at full diversity and enjoys low-complexity ML decoding by employing matched filtering. The main idea of this paper is to enlarge the transmitted signaling set with maximized CGD [1] and full diversity. Then, we enlarge the signaling set based on the STBC Matrix $G(x_1, x_2)$. Two different QPSK constellations proposed by rotating and scaling the conventional constellation are considered. The new constellation can be constructed by multiplying a suitable angle $\exp(j\theta)$ and an amplitude λ. The mathematical equation considered in this paper to modify the QPSK is given by $s' = s \cdot \lambda \exp(j\theta)$, $|\lambda| \le 1, |\theta| \le \pi$, and $s \in \left\{1/\sqrt{2} \cdot (\pm 1 \pm i)\right\}$. The two different constellations are given in the Fig. 1, in which the outer circle can be expressed the conventional QPSK constellation belonged to signal set.1 s and the inner circle can be expressed as the modified signal constellation s'. A very important criterion will be greatly discussed and calculated in this paper derived in [2] $\det((C_i - C_j)'(C_i - C_j))$, and the difference of two codes can be defined as $C_i - C_j = D_{ij}$, here the entrance of G_i belongs to constellation s, and each the entrance of $G_j^{'}$ belongs to s', then we briefly prove that $\det(D_{ij}^{'} D_{ij})$ be full diversity,

$$\det(D_{ij}^{'} D_{ij}) = \det\left(\begin{pmatrix} x_1(1-\lambda e^{j\theta}) & x_2(1-\lambda e^{j\theta}) \\ -x_2^*(1-\lambda e^{-j\theta}) & x_1^*(1-\lambda e^{-j\theta}) \end{pmatrix}^2\right)$$
$$= \det\left(\begin{pmatrix} x_1(1-\lambda e^{j\theta}) & x_2(1-\lambda e^{j\theta}) \\ -x_2^*(1-\lambda e^{-j\theta}) & x_1^*(1-\lambda e^{-j\theta}) \end{pmatrix}\right)^2 \quad (2)$$

while in terms of suitable angle θ and amplitude λ, the $C_i - C_j = D_{ij}$ is full diversity.

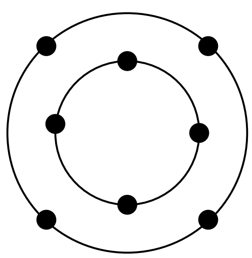

Fig. 1 Conventional and proposed constellations

Then, G_0 and G_1 are given here, the entrances of G_0 are selected from conventional QPSK constellation and the entrances of G_1 are selected from proposed constellation for the transmission scheme. We will use the expanded set S to construct new high-rate (>1) full diversity space-time block code with low decoding complexity and optimized available coding gain.

$$G_0(x_1, x_2, b_0 = 0) = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix},$$
$$G_1(x_1, x_2, b_0 = 1) = \begin{bmatrix} x_1\lambda e^{j\theta} & x_2\lambda e^{j\theta} \\ -x_2^*\lambda e^{j\theta} & x_1^*\lambda e^{j\theta} \end{bmatrix}, \quad (3)$$

We will use the expanded set S to construct new high-rate (> 1) and full diversity space-time block code with low complexity decoding and optimized available coding gain. The space-time code G_0 is selected to transmit symbols while the additional bit b_0 is 0 to be transmitted. With this idea, G_1 is selected for transmitting $b_0 = 1$. Then, each transmission frame consists of two symbols by employing two different constellations with 5 information bits. Without additional system source, the transmission can be improved according such transmission scheme.

III. THE PROPOSED CONSTELLATION FOR TWO ANTENNAS

A. *Design Criterions for the Proposed Constellation*

Consider two distinct codewords $C_i, C_j \in C'$. In order to ensure full spatial diversity, the codeword difference matrix $C_i - C_j = D_{ij}$ between any two distinct codewords in the extended set C must be full rank [4]. When both codewords C_i and C_j belong to G_i or G_j, D_{ii} will be full rank. However if $C_i \in G_i$ and $C_j \in G_{j \ne i}$, D_{ij} loses rank property. To restore full-diversity, schemes based on rotations of information symbols have been proposed (see e.g. [3], [7]). In this paper, we propose to rotate constellation and scale amplitude transmitted signals in s' by a suitable angle θ and λ to guarantee full-diversity for maximizing the CGD. For a unit-radius QPSK constellation, the rotating result is the same with signal constellation for transmission. Consider two different codewords $C_i, C_j \in C$. In order to ensure full spatial diversity, the difference matrix $C_i - C_j = D_{ij}$ between any two distinct codewords in the extend set S must be full rank [2]. When both codewords C_i and C_j belong to the G_0 or G_1, D_{ij} will be full rank. However, if $C_i \in G_0$ and $C_j \in G_1$ (or vice versa), D_{ij} lose full rank property. To restore full-diversity, the scheme based on rotations of information symbols has been proposed [4]. For a unit-radius QPSK constellation, the different angle should guarantee that the proposed constellation is different from the traditional constellation. Then, the coding gain matrices between two different code works should be full rank to achieve the higher determinant as large as possible. The optimum combination of angle θ and λ should follow such rule to design the constellations.

B. *Optimum Values of Angle θ and Amplitude λ*

With different angle θ and λ values, the system performance should be different. The main idea to introducing the angle θ and λ is to ensure full diversity for the proposed high-rate STBC. One important selection criteria for θ and λ is to maximize the determinate of the CG with different combinations of θ and λ. In addition, CG is defined as the minimum product of nonzero singular values of D_{ij} for overall distinct codewords pairs. In order to ensure full spatial diversity, the codeword difference matrix

$\det\left(\left(C_i - C_j\right)^* \left(C_i - C_j\right)\right)$ between any two distinct codewords in the extended set S must be full rank.

$$\det\left(\left(C_i - C_j\right)^* \left(C_i - C_j\right)\right) \hat{=} \det\left(\begin{pmatrix} 1-\lambda e^{j\theta} & 1-\lambda e^{j\theta} \\ 1-\lambda e^{-j\theta} & 1-\lambda e^{-j\theta} \end{pmatrix}^* \begin{pmatrix} 1-\lambda e^{j\theta} & 1-\lambda e^{j\theta} \\ 1-\lambda e^{-j\theta} & 1-\lambda e^{-j\theta} \end{pmatrix}\right) \quad (4)$$

While difference matrix $C_i - C_j = D_{ij}$ is a full rank matrix, $\det(D_{ij} D_{ij}^*) \neq 0$. The optimum value of angle θ and λ while $\det(D_{ij} D_{ij}^*)$ is maximized by using the first derivative property.

$$\begin{cases} \dfrac{\mathrm{d}\left(\det(\mathrm{D_{ij}} \mathrm{D_{ij}^*})\right)}{\mathrm{d}\theta} = 0 \\ \dfrac{\mathrm{d}\left(\det(\mathrm{D_{ij}} \mathrm{D_{ij}^*})\right)}{\mathrm{d}\lambda} = 0 \end{cases} \Rightarrow \begin{cases} \lambda(\sin\theta - \cos\theta) = 0 \\ \lambda^2 - \lambda \sin\theta = 0 \end{cases}, \begin{cases} \theta = \pi/4 \\ \lambda = 1/\sqrt{2} \end{cases} \quad (5)$$

Here, the optimum value of angle θ and λ are derived from these formulas. The candidate of QPSK signal constellation can be obtained by the optimum values of angle θ and λ. Then, the traditional QPSK signal constellation is the collection $s \in \left\{1/\sqrt{2} \cdot (\pm 1 \pm i)\right\}$, and the proposed constellation also exits in another collection $s' \in \lambda s \exp(j\theta) = \left\{\pm 1/2, \pm i\right\}$.

C. Low Complexity Decoding Algorithm

The output symbols $R = \left[r_1, r_2\right]^T$ received vector over two consecutive symbol periods can be represented as follows:

$$\mathbf{R} = HS_{G_i} + N \ , \ \mathbf{H} = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix} \quad (6)$$

where $\mathbf{R}$ are 1×2 complex matrix representations of the received signals corresponding to the transmitted codewords in the form of G_i and $G_{j \neq i}$, respectively. The path gains from the two transmit antennas to the mobile is $\mathbf{H}$ that are independent and identically distributed (i.i.d) with $h_i \sim CN(0, \sigma^2)$ and $E(h_i h_j) = 0$ if $i \neq j$. The channel is assumed to be known perfectly at the receiver and the noise samples $N = [N_1, N_2]^T$ are independent samples of a zero-mean complex Gaussian random variable. The channel matrix $\mathbf{H}$ is a quaternion and we have $HH^* = \left(|h_1|^2 + |h_2|^2\right) I_2$. Two simple matched-filtering operations, $H^* G_i$ and $H^* G_{i \neq j}$ are performed to generate two candidate solutions, namely, $\hat{S}_i$ and $\hat{S}_{j \neq i}$ which are then compared using the metric

$$\tilde{S} = \arg\min_{S} \left\| \left[r_1, r_2\right]^T - HS \right\|^2 \quad (7)$$

The decoding result for $b0$ follows directly once the decision between S_i or $S_{j \neq i}$ is made. Now we begin to state the decoding algorithm for the proposed high-rate scheme. At first, the transmitted codewords can be divided into two parts of S, which can be expressed as mathematical expression $S = \left\{S_1, S_2\right\}$. These two different subsets S_1, S_2 within the set can be used to transmit $G_{1,i} \in S_1, \forall i$ and $G_{2,i} \in S_2, \forall i$ with the additional bit for transmission, respectively. The decoding algorithm can be divided into three steps by using (7)

1. Following the decoding criterion formula (7), one constellation of set S_1, S_2 is selected to decode the transmitted codes through the received vector. Without loss of generality, at the first step, S_1 is selected to decode and note the metric of the decoding result and $d_1 = \left\| \left[r_1, r_2\right]^T - H\tilde{G}_1 \right\|^2$, $\tilde{G}_1$ is the decoding result in our decoding procedure.
$$\tilde{G}_1 = \arg\min_{S_1} \left\| \left[r_1, r_2\right]^T - HS_1 \right\|^2$$
2. At this step, another subset S_2 is selected to decode the transmitted codes as the same procedure with step 1, and note the metric of this step $d_2 = \left\| \left[r_1, r_2\right]^T - H\tilde{G}_2 \right\|^2$
3. To compare with the metrics of previous two steps and select the maximum value between two metrics, the transmitted codes and the additional bit can be obtained through the three steps.

Comments: Although the proposed algorithm can improve the system transmission rate, the decoding algorithm should be more complicated then the original algorithm with more adders and multipliers achieving high rate property.

IV. EXTENTION TO 4 ANTENNAS CASE

Consider the following example of a candidate of the STBC with rate-1 and full-diversity complex quasi-orthogonal design based on Quasi-orthogonal STBC proposed by Jafarkhani [5]. In our proposition, we extend the set of transmitted signal constellation by a multiplied factor of $\lambda \exp(j\theta)$ to the quasi-orthogonal entrance selected from the conventional signal constellation.

$$G = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix} \quad (8)$$

We expand the signaling set and increase the rate to $9/8$ (for QPSK) by considering the following signal multiplied by the multiplying the factor $\lambda\exp(j\theta)$.

$$G_0(x_1,x_2,x_3,x_4,b_0=0)=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix} \qquad (9)$$

$$G_1(x_1,x_2,x_3,x_4,b_0=1)=\begin{bmatrix} x_1e^{j\theta} & x_2e^{j\theta} & x_3e^{j\theta} & x_4e^{j\theta} \\ -x_2^*e^{-j\theta} & x_1^*e^{-j\theta} & -x_4^*e^{-j\theta} & x_3^*e^{-j\theta} \\ -x_3^*e^{-j\theta} & -x_4^*e^{-j\theta} & x_1^*e^{-j\theta} & x_2^*e^{-j\theta} \\ x_4e^{j\theta} & -x_3e^{j\theta} & -x_2e^{j\theta} & x_1e^{j\theta} \end{bmatrix}$$

To compare these different STBCs, the transmission scheme is almost the same with the two antennas case with different rotating angles about the original signal constellation.

The CGD (coding gain distance) between a pair of codewords $C=G(s_1,s_2,s_3,s_4)$ and $C'=G(s'_1,s'_2,s'_3,s'_4)$ from the QOSTBC is given by

$$\begin{aligned} &\mathrm{CGD}(C,C')=\det\left[D(C,C')^H D(C,C')\right] \\ &=(\left|(s_1-s'_1)-(s_4-s'_4)\right|^2+\left|(s_2-s'_2)+(s_3-s'_3)\right|^2)^2 \qquad (10) \\ &\cdot(\left|(s_1-s'_1)+(s_4-s'_4)\right|^2+\left|(s_2-s'_2)-(s_3-s'_3)\right|^2)^2 \end{aligned}$$

Then, in order to simplify these formulas, we replace x_1 with s_1-s'_1, x_2 with s_2-s'_2, x_3 with s_3-s'_3 ,and x_4 with s_4-s'_4

$$\det(G^HG)=(|x_1-x_4|^2+|x_2+x_3|^2)^2(|x_1+x_4|^2+|x_2-x_3|^2)^2 \quad (11)$$

To derive the optimum values of angle θ and λ, the simpler first derivative of $\det(G^HG)$ also calculates.

$$\begin{cases} \dfrac{d(\det(G^HG))}{d\lambda}=\lambda^3(\cos^4\theta-\sin^2\theta\cos^2\theta) \\ \dfrac{d(\det(G^HG))}{d\theta}=3\lambda^4\cos^3\theta\sin\theta-\lambda^2\sin\cos^2\theta \end{cases} \qquad (12)$$

$$\Rightarrow \theta=\pi, \lambda=1/\sqrt{3}$$

Here, a similar way can be used to calculate the optimum values of angle θ and λ, the values derived are used to construct the new transmitted signal constellation according our proposed motivation in this paper.

V. NUMERICAL RESULTS

In this section, we show some simulation results for the proposed transmission scheme. Fig. 2 shows the BER performances versus SNR to compare the proposed design with the Almouti code and Quasi-orthogonal [5] equipped with 2 or 4 transmitting and receiving antennas. The proposed transmission scheme simulated in this figure has a similar performance with Quasi-orthogonal above 20dB. QPSK modulation, 8PSK modulation and flat fading channel are considered in all simulations. In the high SNR region 20-25 dB of Fig. 1, the BER performance of proposed scheme with a high rate-9/8 transmission is similar to Quasi-orthogonal code, but there also exists a visible gap in the low SNR region 10-20dB. Moreover, the BER performance of proposed design is almost 1-2dB away from the conventional Almouti code and Quasi-orthogonal at the BER of 10^{-5}.

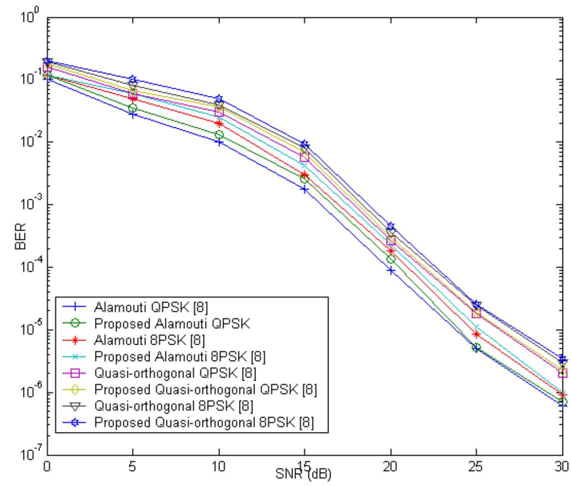

Fig. 2 Comparisons of FER Performance

In Fig. 3, we compare our proposed rate- 5/4 code with the Alamouti [1] code and rate- 9/8 code with the Quasi-orthogonal code [6] using the measure of Effective Throughput η defined as:

$$\eta=(1-FER)*R*\log_2(M), \qquad (13)$$

where R is the code rate, and M is the constellation size, and FER denotes the frame error rate, and means the frame error rate, and each frame contains 4 symbols (9 bits plus one additional bit). The Fig. 3 shows that at high SNR, our code achieves a higher throughput level of channel use (PCU) whereas the achievable throughput for the Alamouti code [1] PCU. Fig. 3 also depicts the achievable throughput of a $rate-9/8$ code that uses pure rotations to ensure full-diversity and maximize coding gain. The proposed transmission scheme achieves 2.25 bits per channel transmission whereas the effective throughput of the QOSTBC is 2 bits. A crossing point exits at a SNR level of 20dB. Similarly, the effective throughput performance of high rate-9/8 [8] is also simulated in this figure. From 20dB to 30dB of SNR, the effective throughput has been improved compared to [8] due to low FER in the proposed design. This result is matched with the conclusion of Fig. 1 in which the system performance is affected by the CGD of coding gain matrix.

It demonstrates that optimum rotation achieves higher performance for all values of SNR. The selective angle rotation ensures full-diversity and high rate at the cost of reduced coding gain. It is also possible to turn this coding loss into gain by introducing block codes for the fading channel as in [7]. For an overall rate of 1, this combination of coding techniques outperforms the Alamouti code [1] at the price of higher decoding complexity.

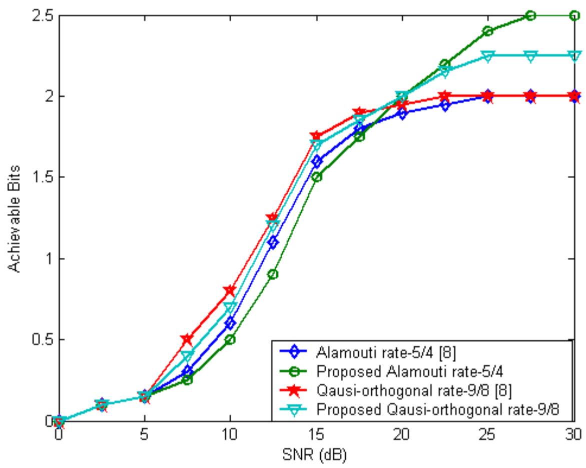

Fig. 3 Comparisons of Throughput Performance

VI. CONCLUSIONS

We exploited the algebraic structure of quaternions to design and optimize a novel high-rate, full-diversity STBC for 2 and 4 transmitting antennas. We introduced the concept of selective power scaling to guarantee full diversity for the designed code. The angle rotation was further optimized to maximize available coding gain. The computer simulations show that the effective throughput is improved in the high SNR region.

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